

16. On January 1, 1997, an investment account is worth 100,000 . On April 1, 1997, the value has increased to 103,000 and 8,000 is withdrawn. On January 1, 1999, the account is worth 103,992 .

Assuming a dollar weighted method for 1997 and a time weighted method for 1998, the annual effective interest rate was equal to x for both 1997 and 1998 .

Calculate x .

- (A) 6.00%
- (B) 6.25%
- (C) 6.50%
- (D) 6.75%
- (E) 7.00%

51. Seth deposits X in an account today in order to fund his retirement. He would like to receive payments of 50 per year, in real terms, at the end of each year for a total of 12 years, with the first payment occurring seven years from now.

The inflation rate will be 0.0% for the next six years and 1.2% per annum thereafter.

The annual effective rate of return is 6.3% .

Calculate X .

- (A) 303
- (B) 306
- (C) 316
- (D) 327
- (E) 329

2. The following table shows the annual effective interest rates being credited by an investment account, by calendar year of investment. The investment year method is applicable for the first 3 years, after which a portfolio rate is used:

Calendar Year of Investment	Investment Year Rates			Calendar Year of Portfolio Rate	Portfolio Rate
	i_1	i_2	i_3		
1990	10%	10%	$t\%$	1993	8%
1991	12%	5%	10%	1994	$(t-1)\%$
1992	8%	$(t-2)\%$	12%	1995	6%
1993	9%	11%	6%	1996	9%
1994	7%	7%	10%	1997	10%

An investment of 100 is made at the beginning of years 1990, 1991, and 1992 . The total amount of interest credited by the fund during the year 1993 is equal to 28.40 .

Calculate t .

- (A) 7.00
- (B) 7.25
- (C) 7.50
- (D) 7.75
- (E) 8.00

27. An investor deposits 50 in an investment account on January 1 . The following summarizes the activity in the account during the year:

<u>Date</u>	<u>Value Immediately Before Deposit</u>	<u>Deposit</u>
March 15	40	20
June 1	80	80
October 1	175	75

On June 30, the value of the account is 157.50 . On December 31, the value of the account is X . Using the time-weighted method, the equivalent annual effective yield during the first 6 months is equal to the (time-weighted) annual effective yield during the entire 1-year period.

Calculate X .

- (A) 234.75
- (B) 235.50
- (C) 236.25
- (D) 237.00
- (E) 237.75

38. Chuck needs to purchase an item in 10 years. The item costs 200 today, but its price inflates 4% per year.

To finance the purchase, Chuck deposits 20 into an account at the beginning of each year for 6 years. He deposits an additional X at the beginning of years 4, 5, and 6 to meet his goal.

The annual effective interest rate is 10% .

Calculate X .

- (A) 7.4
- (B) 7.9
- (C) 8.4
- (D) 8.9
- (E) 9.4

31. You are given the following information about an investment account:

Date	Value Immediately Before Deposit	Deposit
January 1	10	
July 1	12	X
December 31	X	

Over the year, the time-weighted return is 0%, and the dollar-weighted return is Y .

Calculate Y .

- (A) -25%
- (B) -10%
- (C) 0%
- (D) 10%
- (E) 25%

20. You are given the following information about the activity in two different investment accounts:

Account K			
Date	Fund value before activity	Activity	
		Deposit	Withdrawal
January 1, 1999	100.0		
July 1, 1999	125.0		X
October 1, 1999	110.0	$2X$	
December 31, 1999	125.0		

Account L			
Date	Fund value before activity	Activity	
		Deposit	Withdrawal
January 1, 1999	100.0		
July 1, 1999	125.0		X
December 31, 1999	105.8		

During 1999, the dollar weighted return for investment account K equals the time weighted return for investment account L, which equals i .

Calculate i .

- (A) 10%
- (B) 12%
- (C) 15%
- (D) 18%
- (E) 20%

17. An association had a fund balance of 75 on January 1 and 60 on December 31. At the end of every month during the year, the association deposited 10 from membership fees. There were withdrawals of 5 on February 28, 25 on June 30, 80 on October 15, and 35 on October 31.

Calculate the dollar-weighted rate of return for the year.

- (A) 9.0%
- (B) 9.5%
- (C) 10.0%
- (D) 10.5%
- (E) 11.0%

30. You are given the following table of interest rates:

Calendar Year of Original Investment	Investment Year Rates (in %)					Portfolio Rates (in %)
	i_1^y	i_2^y	i_3^y	i_4^y	i_5^y	
1992	8.25	8.25	8.4	8.5	8.5	8.35
1993	8.5	8.7	8.75	8.9	9.0	8.6
1994	9.0	9.0	9.1	9.1	9.2	8.85
1995	9.0	9.1	9.2	9.3	9.4	9.1
1996	9.25	9.35	9.5	9.55	9.6	9.35
1997	9.5	9.5	9.6	9.7	9.7	
1998	10.0	10.0	9.9	9.8		
1999	10.0	9.8	9.7			
2000	9.5	9.5				
2001	9.0					

A person deposits 1000 on January 1, 1997. Let the following be the accumulated value of the 1000 on January 1, 2000:

- P : under the investment year method
 Q : under the portfolio yield method
 R : where the balance is withdrawn at the end of every year and is reinvested at the new money rate

Determine the ranking of P , Q , and R .

- (A) $P > Q > R$
 (B) $P > R > Q$
 (C) $Q > P > R$
 (D) $R > P > Q$
 (E) $R > Q > P$

3. A bond will pay a coupon of 100 at the end of each of the next three years and will pay the face value of 1000 at the end of the three-year period. The bond's duration (Macaulay duration) when valued using an annual effective interest rate of 20% is X .

Calculate X .

- (A) 2.61
- (B) 2.70
- (C) 2.77
- (D) 2.89
- (E) 3.00

6. John purchased three bonds to form a portfolio as follows:

Bond A has semi-annual coupons at 4%, a duration of 21.46 years, and was purchased for 980.

Bond B is a 15-year bond with a duration of 12.35 years and was purchased for 1015.

Bond C has a duration of 16.67 years and was purchased for 1000.

Calculate the duration of the portfolio at the time of purchase.

- (A) 16.62 years
- (B) 16.67 years
- (C) 16.72 years
- (D) 16.77 years
- (E) 16.82 years

10. Yield rates to maturity for zero coupon bonds are currently quoted at 8.5% for one-year maturity, 9.5% for two-year maturity, and 10.5% for three-year maturity. Let i be the one-year forward rate for year two implied by current yields of these bonds.

Calculate i .

- (A) 8.5%
- (B) 9.5%
- (C) 10.5%
- (D) 11.5%
- (E) 12.5%

15. An insurance company accepts an obligation to pay 10,000 at the end of each year for 2 years. The insurance company purchases a combination of the following two bonds at a total cost of X in order to exactly match its obligation:

- (i) 1-year 4% annual coupon bond with a yield rate of 5%
- (ii) 2-year 6% annual coupon bond with a yield rate of 5%.

Calculate X .

- (A) 18,564
- (B) 18,574
- (C) 18,584
- (D) 18,594
- (E) 18,604

16. At the beginning of the year, an investment fund was established with an initial deposit of 1000. A new deposit of 1000 was made at the end of 4 months. Withdrawals of 200 and 500 were made at the end of 6 months and 8 months, respectively. The amount in the fund at the end of the year is 1560.

Calculate the dollar-weighted (money-weighted) yield rate earned by the fund during the year.

- (A) 18.57%
- (B) 20.00%
- (C) 22.61%
- (D) 26.00%
- (E) 28.89%

2. Calculate the Macaulay duration of an eight-year 100 par value bond with 10% annual coupons and an effective rate of interest equal to 8%.

- (A) 4
- (B) 5
- (C) 6
- (D) 7
- (E) 8

6. Consider a yield curve defined by the following equation:

$$i_k = 0.09 + 0.002k - 0.001k^2$$

where i_k is the annual effective rate of return for zero coupon bonds with maturity of k years.

Let j be the one-year effective rate during year 5 that is implied by this yield curve.

Calculate j .

- (A) 4.7%
- (B) 5.8%
- (C) 6.6%
- (D) 7.5%
- (E) 8.2%

10. A company must pay liabilities of 1000 and 2000 at the end of years 1 and 2, respectively. The only investments available to the company are the following two zero-coupon bonds:

Maturity (years)	Effective annual yield	Par
1	10%	1000
2	12%	1000

Determine the cost to the company today to match its liabilities exactly.

- (A) 2007
- (B) 2259
- (C) 2503
- (D) 2756
- (E) 3001

15. You are given the following term structure of spot interest rates:

Term (in years)	Spot interest rate
1	5.00%
2	5.75%
3	6.25%
4	6.50%

A three-year annuity-immediate will be issued a year from now with annual payments of 5000.

Using the forward rates, calculate the present value of this annuity a year from now.

- (A) 13,094
- (B) 13,153
- (C) 13,296
- (D) 13,321
- (E) 13,401

21. Which of the following statements about immunization strategies are true?

- I. To achieve immunization, the convexity of the assets must equal the convexity of the liabilities.
- II. The full immunization technique is designed to work for any change in the interest rate.
- III. The theory of immunization was developed to protect against adverse effects created by changes in interest rates.

- (A) None
- (B) I and II only
- (C) I and III only
- (D) II and III only
- (E) The correct answer is not given by (A), (B), (C), and (D).

30.

As of 12/31/2013, an insurance company has a known obligation to pay 1,000,000 on 12/31/2017. To fund this liability, the company immediately purchases 4-year 5% annual coupon bonds totaling 822,703 of par value. The company anticipates reinvestment interest rates to remain constant at 5% through 12/31/2017. The maturity value of the bond equals the par value.

Consider two reinvestment interest rate movement scenarios effective 1/1/2014. Scenario A has interest rates drop by 0.5%. Scenario B has interest rates increase by 0.5%.

Determine which of the following best describes the insurance company's profit or (loss) as of 12/31/2017 after the liability is paid.

- (A) Scenario A – 6,610, Scenario B – 11,150
- (B) Scenario A – (14,760), Scenario B – 14,420
- (C) Scenario A – (18,910), Scenario B – 19,190
- (D) Scenario A – (1,310), Scenario B – 1,320
- (E) Scenario A – 0, Scenario B – 0

31.

An insurance company has an obligation to pay the medical costs for a claimant. Annual claim costs today are 5000, and medical inflation is expected to be 7% per year. The claimant will receive 20 payments.

Claim payments are made at yearly intervals, with the first claim payment to be made one year from today.

Calculate the present value of the obligation using an annual effective interest rate of 5%.

- (A) 87,900
- (B) 102,500
- (C) 114,600
- (D) 122,600
- (E) Cannot be determined

32.

An investor pays 100,000 today for a 4-year investment that returns cash flows of 60,000 at the end of each of years 3 and 4. The cash flows can be reinvested at 4.0% per annum effective.

Using an annual effective interest rate of 5.0%, calculate the net present value of this investment today.

- (A) -1398
- (B) -699
- (C) 699
- (D) 1398
- (E) 2,629

33.

You are given the following information with respect to a bond:

- (i) par value: 1000
- (ii) term to maturity: 3 years
- (iii) annual coupon rate: 6% payable annually

You are also given that the one, two, and three year annual spot interest rates are 7%, 8%, and 9% respectively.

Calculate the value of the bond.

- (A) 906
- (B) 926
- (C) 930
- (D) 950
- (E) 1000

34.

You are given the following information with respect to a bond:

- (i) par value: 1000
- (ii) term to maturity: 3 years
- (iii) annual coupon rate: 6% payable annually

You are also given that the one, two, and three year annual spot interest rates are 7%, 8%, and 9% respectively.

The bond is sold at a price equal to its value.

Calculate the annual effective yield rate for the bond i.

- (A) 8.1%
- (B) 8.3%
- (C) 8.5%
- (D) 8.7%
- (E) 8.9%

35.

The current price of an annual coupon bond is 100. The yield to maturity is an annual effective rate of 8%. The derivative of the price of the bond with respect to the yield to maturity is -700.

Using the bond's yield rate, calculate the Macaulay duration of the bond in years.

- (A) 7.00
- (B) 7.49
- (C) 7.56
- (D) 7.69
- (E) 8.00

36.

A common stock pays a constant dividend at the end of each year into perpetuity.

Using an annual effective interest rate of 10%, calculate the Macaulay duration of the stock.

- (A) 7 years
- (B) 9 years
- (C) 11 years
- (D) 19 years
- (E) 27 years

37.

A common stock pays dividends at the end of each year into perpetuity. Assume that the dividend increases by 2% each year.

Using an annual effective interest rate of 5%, calculate the Macaulay duration of the stock in years.

- (A) 27
- (B) 35
- (C) 44
- (D) 52
- (E) 58

~~38. 44 deleted~~

51.

Joe must pay liabilities of 1,000 due 6 months from now and another 1,000 due one year from now. There are two available investments:

Bond I: A 6-month bond with face amount of 1,000, an 8% nominal annual coupon rate convertible semiannually, and a 6% nominal annual yield rate convertible semiannually;

Bond II: A one-year bond with face amount of 1,000, a 5% nominal annual coupon rate convertible semiannually, and a 7% nominal annual yield rate convertible semiannually.

Calculate the amount of each bond that Joe should purchase to exactly match the liabilities.

- (A) Bond I – 1, Bond II – 0.97561
- (B) Bond I – 0.93809, Bond II – 1
- (C) Bond I – 0.97561, Bond II – 0.94293
- (D) Bond I – 0.93809, Bond II – 0.97561
- (E) Bond I – 0.98345, Bond II – 0.97561

52.

Joe must pay liabilities of 2000 due one year from now and another 1000 due two years from now. He exactly matches his liabilities with the following two investments:

Mortgage I: A one year mortgage in which X is lent. It is repaid with a single payment at time one. The annual effective interest rate is 6%.

Mortgage II: A two-year mortgage in which Y is lent. It is repaid with two equal annual payments. The annual effective interest rate is 7%.

Calculate $X + Y$.

- (A) 2600
- (B) 2682
- (C) 2751
- (D) 2825
- (E) 3000

53.

Joe must pay liabilities of 1,000 due one year from now and another 2,000 due three years from now. There are two available investments:

Bond I: A one-year zero-coupon bond that matures for 1000. The yield rate is 6% per year

Bond II: A two-year zero-coupon bond with face amount of 1,000. The yield rate is 7% per year.

At the present time the one-year forward rate for an investment made two years from now is 6.5%

Joe plans to buy amounts of each bond. He plans to reinvest the proceeds from Bond II in a one-year zero-coupon bond. Assuming the reinvestment earns the forward rate, calculate the total purchase price of Bond I and Bond II where the amounts are selected to exactly match the liabilities.

- (A) 2584
- (B) 2697
- (C) 2801
- (D) 2907
- (E) 3000

54.

Matt purchased a 20-year par value bond with an annual nominal coupon rate of 8% payable semiannually at a price of 1722.25. The bond can be called at par value X on any coupon date starting at the end of year 15 after the coupon is paid. The lowest yield rate that Matt can possibly receive is a nominal annual interest rate of 6% convertible semiannually.

Calculate X .

- (A) 1400
- (B) 1420
- (C) 1440
- (D) 1460
- (E) 1480

57.

Mary purchased a 10-year par value bond with an annual nominal coupon rate of 4% payable semiannually at a price of 1021.50. The bond can be called at 100 over the par value of 1100 on any coupon date starting at the end of year 5 and ending six months prior to maturity.

Calculate the minimum yield that Mary could receive, expressed as an annual nominal rate of interest convertible semiannually.

- (A) 4.7%
- (B) 4.9%
- (C) 5.1%
- (D) 5.3%
- (E) 5.5%

58. Moved to Derivatives Section

59.

A liability consists of a series of 15 annual payments of 35,000 with the first payment to be made one year from now.

The assets available to immunize this liability are five-year and ten-year zero-coupon bonds.

The annual effective interest rate used to value the assets and the liability is 6.2%. The liability has the same present value and duration as the asset portfolio.

Calculate the amount invested in the five-year zero-coupon bonds.

- (A) 127,000
- (B) 167,800
- (C) 208,600
- (D) 247,900
- (E) 292,800

66.

Krishna buys an n -year 1000 bond at par. The Macaulay duration is 7.959 years using an annual effective interest rate of 7.2%.

Calculate the estimated price of the bond, using duration, if the interest rate rises to 8.0%.

- (A) 940.60
- (B) 942.88
- (C) 944.56
- (D) 947.03
- (E) 948.47

67.

The prices of zero-coupon bonds are:

Maturity	Price
1	0.95420
2	0.90703
3	0.85892

Calculate the third year, one-year forward rate.

- (A) 0.048
- (B) 0.050
- (C) 0.052
- (D) 0.054
- (E) 0.056

68.

Sam buys an eight-year, 5000 par bond with an annual coupon rate of 5%, paid annually. The bond sells for 5000. Let d_1 be the Macaulay duration just before the first coupon is paid. Let d_2 be the Macaulay duration just after the first coupon is paid.

Calculate $\frac{d_1}{d_2}$.

- (A) 0.91
- (B) 0.93
- (C) 0.95
- (D) 0.97
- (E) 1.00

69.

An insurance company must pay liabilities of 99 at the end of one year, 102 at the end of two years and 100 at the end of three years. The only investments available to the company are the following three bonds. Bond A and Bond C are annual coupon bonds. Bond B is a zero-coupon bond.

Bond	Maturity (in years)	Yield-to-Maturity (Annualized)	Coupon Rate
A	1	6%	7%
B	2	7%	0%
C	3	9%	5%

All three bonds have a par value of 100 and will be redeemed at par.

Calculate the number of units of Bond A that must be purchased to match the liabilities exactly.

- (A) 0.8807
- (B) 0.8901
- (C) 0.8975
- (D) 0.9524
- (E) 0.9724

70.

Determine which of the following statements is false with respect to Redington immunization.

- (A) Modified duration may change at different rates for each of the assets and liabilities as time goes by.
- (B) Redington immunization requires infrequent rebalancing to keep modified duration of assets equal to modified duration of liabilities.
- (C) This technique is designed to work only for small changes in the interest rate.
- (D) The yield curve is assumed to be flat.
- (E) The yield curve shifts in parallel when the interest rate changes.

71.

Aakash has a liability of 6000 due in four years. This liability will be met with payments of A in two years and B in six years. Aakash is employing a full immunization strategy using an annual effective interest rate of 5%.

Calculate $|A - B|$.

- (A) 0
- (B) 146
- (C) 293
- (D) 586
- (E) 881

72.

Jia Wen has a liability of 12,000 due in eight years. This liability will be met with payments of 5000 in five years and B in $8 + b$ years. Jia Wen is employing a full immunization strategy using an annual effective interest rate of 3%.

Calculate $\frac{B}{b}$.

- (A) 2807
- (B) 2873
- (C) 2902
- (D) 2976
- (E) 3019

73.

Trevor has assets at time 2 of A and at time 9 of B . He has a liability of 95,000 at time 5. Trevor has achieved Redington immunization in his portfolio using an annual effective interest rate of 4%.

Calculate $\frac{A}{B}$.

- (A) 0.7307
- (B) 0.9670
- (C) 1.0000
- (D) 1.0132
- (E) 1.3686

78.

On January 1, an investment fund was opened with an initial balance of 5000. Just after the balance grew to 5200 on July 1, an additional 2600 was deposited.

The annual effective yield rate for this fund was 9.00% over the calendar year.

Calculate the time-weighted rate of return for the year.

- (A) 7.43%
- (B) 8.86%
- (C) 9.00%
- (D) 9.17%
- (E) 10.45%

79.

Bill and Joe each put 10 into separate accounts at time $t = 0$, where t is measured in years. Bill's account earns interest at a constant annual effective interest rate of $K/25$, $K > 0$.

Joe's account earns interest at a force of interest, $\delta_t = \frac{K}{25} \sqrt{t}$.

At the end of four years, the amount in each account is X .

Calculate X .

- (A) 20.7
- (B) 21.7
- (C) 22.7
- (D) 23.7
- (E) 24.7

82.

You are given:

- (i) F = the amount in a fund at the beginning of the year
- (ii) There are n cash flows during the year, where each cash flow, $C_k, k=1, 2, \dots, n$ is made at time $t_k, 0 < t \leq 1$.
- (iii) I = the amount of interest earned during the year.

Consider the formula

$$I = F + \sum_{k=1}^n C_k (1 - t_k)$$

Determine which of the following conditions produces i closest to the annual effective yield rate

- (A) When each cash flow is small relative to F
- (B) When each cash flow is large relative to F
- (C) When each cash flow is large relative to I
- (D) When the cash flows all have the same sign
- (E) When the timing of cash flows is uniform throughout the year

83.

On January 1, a fund is worth 100,000. On May 1, the value has increased to 120,000 and then 30,000 of new principal is deposited. On November 1, the value has declined to 130,000 and then 50,000 is withdrawn. On January 1 of the following year, the fund is again worth 100,000.

Calculate the time-weighted rate of return.

- (A) 0.00%
- (B) 17.91%
- (C) 25.00%
- (D) 29.27%
- (E) 30.00%

83.

88.

A borrower takes out a 15-year loan for 65,000, with level end-of-month payments. The annual nominal interest rate of the loan is 8%, convertible monthly.

Immediately after the 12th payment is made, the remaining loan balance is reamortized. The maturity date of the loan remains unchanged, but the annual nominal interest rate of the loan is changed to 6%, convertible monthly.

Calculate the new end-of-month payment.

- (A) 528
- (B) 534
- (C) 540
- (D) 546
- (E) 552

89.

College tuition is 6000 for the current school year, payable in full at the beginning of the school year. College tuition will grow at an annual rate of 5%. A parent sets up a college savings fund earning interest at an annual effective rate of 7%. The parent deposits 750 at the beginning of each school year for 18 years, with the first deposit made at the beginning of the current school year. Immediately following the 18th deposit, the parent pays tuition for the 18th school year from the fund.

The amount of money needed, in addition to the balance in the fund, to pay tuition at the beginning of the 19th school year is X .

Calculate X .

- (A) 1439
- (B) 1545
- (C) 1664
- (D) 1785
- (E) 1870

92.

You are given the following term structure of interest rates:

Length of investment in years	Spot rate
1	7.50%
2	8.00%
3	8.50%
4	9.00%
5	9.50%
6	10.00%

Calculate the one-year forward rate for the fifth year implied by this term structure.

- (A) 9.5%
- (B) 10.0%
- (C) 11.5%
- (D) 12.0%
- (E) 12.5%

93.

Seth has two retirement benefit options.

His first option is to receive a lump sum of 374,500 at retirement.

His second option is to receive monthly payments for 25 years starting one month after retirement. For the first year, the amount of each monthly payment is 2000. For each subsequent year, the monthly payments are 2% more than the monthly payments from the previous year.

Using an annual nominal interest rate of 6%, compounded monthly, the present value of the second option is P .

Determine which of the following is true.

- (A) P is 323,440 more than the lump sum option amount.
- (B) P is 107,170 more than the lump sum option amount.
- (C) The lump sum option amount is equal to P .
- (D) The lump sum option amount is 60 more than P .
- (E) The lump sum option amount is 64,090 more than P .

94

110.

A 5-year loan of 300,000 with an annual effective discount rate of 8% is to be repaid by level end-of-year payments.

If the first four payments had been rounded up to the next multiple of 1,000, the final payment would be X .

Calculate X .

- (A) 103,500
- (B) 111,700
- (C) 115,200
- (D) 125,200
- (E) 127,500

111.

A company plans to invest X at the beginning of each month in a zero-coupon bond in order to accumulate 100,000 at the end of six months. The price of each bond as a percentage of redemption value is given in the following chart:

Maturity (months)	1	2	3	4	5	6
Price	99%	98%	97%	96%	95%	94%

Calculate X given that the bond prices will not change during the six-month period.

- (A) 15,667
- (B) 16,078
- (C) 16,245
- (D) 16,667
- (E) 17,271

118.

An n -year bond with semiannual coupons has the following characteristics:

- i) The par value and redemption value are 2500;
- ii) The annual coupon rate is 7% payable semi-annually;
- iii) The annual nominal yield to maturity is 8% convertible semiannually; and
- iv) The book value immediately after the fourth coupon is 8.44 greater than the book value immediately after the third coupon.

Calculate n .

- (A) 6.5
- (B) 7.0
- (C) 9.5
- (D) 12.0
- (E) 14.0

119.

For the next four years, the one-year forward rates of interest are estimated to be:

Year	0	1	2	3	4
Forward Rate	4%	6%	8%	10%	12%

Calculate the spot rate for a zero-coupon bond maturing three years from now.

- (A) 4%
- (B) 5%
- (C) 6%
- (D) 7%
- (E) 8%

120.

On January 1, an investment account is worth 50,000. On May 1, the value has increased to 52,000 and 8,000 of new principal is deposited. At time t , in years, ($4/12 < t < 1$) the value of the fund has increased to 62,000 and 10,000 is withdrawn. On January 1 of the next year, the investment account is worth 55,000. The dollar-weighted rate of return (using the simple interest approximation) is equal to the time-weighted rate of return for the year.

Calculate t .

- (A) 0.411
- (B) 0.415
- (C) 0.585
- (D) 0.589
- (E) 0.855

121.

Annuity A pays 1 at the beginning of each year for three years. Annuity B pays 1 at the beginning of each year for four years.

The Macaulay duration of Annuity A at the time of purchase is 0.93. Both annuities offer the same yield rate.

Calculate the Macaulay duration of Annuity B at the time of purchase.

- (A) 1.240
- (B) 1.369
- (C) 1.500
- (D) 1.930
- (E) 1.965

122.

Cash flows are 40,000 at time 2 (in years), 25,000 at time 3, and 100,000 at time 4. The annual effective yield rate is 7.0%.

Calculate the Macaulay duration.

- (A) 2.2
- (B) 2.3
- (C) 3.1
- (D) 3.3
- (E) 3.4

123.

Bond A is a 3-year bond that pays annual coupons and is priced at par. The annual coupon rate is 10%.

Bond B pays semiannual coupons and yields 10% convertible semiannually.

Bond A and Bond B have the same modified duration.

Calculate the Macaulay duration of Bond B.

- (A) 2.49
- (B) 2.61
- (C) 2.74
- (D) 2.87
- (E) 3.01

124.

Rhonda purchases a perpetuity providing a payment of 1 at the beginning of each year. The perpetuity's Macaulay duration is 30 years.

Calculate the modified duration of this perpetuity.

- (A) 28.97
- (B) 29.00
- (C) 29.03
- (D) 29.07
- (E) 29.10

125.

Stocks F and J are valued using the dividend discount model. The required annual effective rate of return is 8.8%. The dividend of Stock F has an annual growth rate of g and the dividend of Stock J has an annual growth rate of $-g$.

The dividends of both stocks are paid annually on the same date.

The value of Stock F is twice the value of Stock J. The next dividend on Stock F is half of the next dividend on Stock J.

Calculate g .

- (A) 0.0%
- (B) 0.8%
- (C) 2.9%
- (D) 5.3%
- (E) 8.8%

126.

Which of the following statements regarding immunization are true?

- I. If long-term interest rates are lower than short-term rates, the need for immunization is reduced.
- II. Either Macaulay or modified duration can be used to develop an immunization strategy.
- III. Both processes of matching the present values of the flows or the flows themselves will produce exact matching.

- (A) I only
- (B) II only
- (C) III only
- (D) I, II and III
- (E) The correct answer is not given by (A), (B), (C), or (D).

127.

A company owes 500 and 1000 to be paid at the end of year one and year four, respectively. The company will set up an investment program to match the duration and the present value of the above obligation using an annual effective interest rate of 10%.

The investment program produces asset cash flows of X today and Y in three years.

Calculate X and determine whether the investment program satisfies the conditions for Redington immunization.

- (A) $X = 75$ and the Redington immunization conditions are not satisfied.
- (B) $X = 75$ and the Redington immunization conditions are satisfied.
- (C) $X = 1138$ and the Redington immunization conditions are not satisfied.
- (D) $X = 1138$ and the Redington immunization conditions are satisfied.
- (E) $X = 1414$ and the Redington immunization conditions are satisfied.

128.

An insurance company has a known liability of 1,000,000 that is due 8 years from now. The technique of full immunization is to be employed. Asset I will provide a cash flow of 300,000 exactly 6 years from now. Asset II will provide a cash flow of X , exactly y years from now, where $y > 8$.

The annual effective interest rate is 4%.

Calculate X .

- (A) 697,100
- (B) 698,600
- (C) 700,000
- (D) 701,500
- (E) 702,900

129.

A company has liabilities of 573 due at the end of year 2 and 701 due at the end of year 5.

A portfolio comprises two zero-coupon bonds, Bond A and Bond B.

Determine which portfolio produces a Redington immunization of the liabilities using an annual effective interest rate of 7.0%.

- (A) Bond A: 1-year, current price 500; Bond B: 6-years, current price 500
- (B) Bond A: 1-year, current price 572; Bond B: 6-years, current price 428
- (C) Bond A: 3-years, current price 182; Bond B: 4-years, current price 1092
- (D) Bond A: 3-years, current price 637; Bond B: 4-years, current price 637
- (E) Bond A: 3.5 years, current price 1000; Bond B: Not used

130.

A company has liabilities of 402.11 due at the end of each of the next three years. The company will invest 1000 today to fund these payouts. The only investments available are one-year and three-year zero-coupon bonds, and the yield curve is flat at a 10% annual effective rate. The company wishes to match the duration of its assets to the duration of its liabilities.

Determine how much the company should invest in each bond.

- (A) 366 in the one-year bond and 634 in the three-year bond.
- (B) 484 in the one-year bond and 516 in the three-year bond.
- (C) 500 in the one-year bond and 500 in the three-year bond.
- (D) 532 in the one-year bond and 468 in the three-year bond.
- (E) 634 in the one-year bond and 366 in the three-year bond.

131.

You are given the following information about a company's liabilities:

- Present value: 9697
- Macaulay duration: 15.24
- Macaulay convexity: 242.47

The company decides to create an investment portfolio by making investments into two of the following three zero-coupon bonds: 5-year, 15-year, and 20-year. The company would like its position to be Redington immunized against small changes in yield rate.

The annual effective yield rate for each of the bonds is 7.5%.

Determine which of the following portfolios the company should create.

- (A) Invest 3077 for the 5-year bond and 6620 for the 20-year bond.
- (B) Invest 6620 for the 5-year bond and 3077 for the 20-year bond.
- (C) Invest 465 for the 15-year bond and 9232 for the 20-year bond.
- (D) Invest 4156 for the 15-year bond and 5541 for the 20-year bond.
- (E) Invest 9232 for the 15-year bond and 465 for the 20-year bond.

132.

A bank accepts a 20,000 deposit from a customer on which it guarantees to pay an annual effective interest rate of 10% for two years. The customer needs to withdraw half of the accumulated value at the end of the first year. The customer will withdraw the remaining value at the end of the second year.

The bank has the following investment options available, which may be purchased in any quantity:

- Bond H: A one-year zero-coupon bond yielding 10% annually
- Bond I: A two-year zero-coupon bond yielding 11% annually
- Bond J: A two-year bond that sells at par with 12% annual coupons

Any portion of the 20,000 deposit that is not needed to be invested in bonds is retained by the bank as profit.

Determine which of the following investment strategies produces the highest profit for the bank and is guaranteed to meet the customer's withdrawal needs.

- (A) 9,091 in Bond H, 8,264 in Bond I, 2,145 in Bond J
- (B) 10,000 in Bond H, 10,000 in Bond I
- (C) 10,000 in Bond H, 9,821 in Bond I
- (D) 8,910 in Bond H, 731 in Bond I, 10,000 in Bond J
- (E) 8,821 in Bond H, 10,804 in Bond J

133.

An insurance company wants to match liabilities of 25,000 payable in one year and 20,000 payable in two years with specific assets. The following assets are currently available:

- i) One-year bond with an annual coupon of 6.75% at par
- ii) Two-year bond with annual coupons of 4.50% at par
- iii) Two-year zero-coupon bond yielding 5.00% annual effective

Calculate the smallest amount the company needs to disburse today to purchase assets that will exactly match these liabilities.

- (A) 41,220
- (B) 41,390
- (C) 41,560
- (D) 41,660
- (E) 41,750